CE 440 Introduction to Operating System

Lecture 12: Page Replacement
Fall 2025

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Slides courtesy of Manuel Egele, Ryan Huang and Baris Kasikci

Administrivia

- Next next Wednesday (October 22nd) is Midterm
- Midterm Review Session on next Monday

Memory Management

Next four lectures are going to cover memory management

Goals of memory management

Mechanisms

- Physical and virtual addressing (1)
- Techniques: partitioning, paging, segmentation (1)
- Page table management, TLBs, VM tricks (2)

Policies

- Page replacement algorithms (3)

Lecture Overview

Review paging and page replacement

Page replacement algorithms

Local vs. global replacement

Thrashing

Recap: Demand Paging

Paging (swapping) from the OS perspective:

- Pages are evicted to disk when memory is full
- Pages loaded from disk when referenced again
- References to evicted pages cause a TLB miss
    - PTE was invalid, causes fault
- OS allocates a page frame, reads page from disk to the page frame
- OS fills in PTE, marks it valid, and restarts faulting instruction

Dirty vs. clean pages

- Actually, only dirty pages (modified) need to be written to disk
- Clean pages do not
    - but you do need to know where on disk to read them from again!

Paging Challenges

How to resume a process after a fault?

- Need to save state and resume

What to fetch from disk?

- Just needed page or more?

What to evict?

- How to allocate physical pages amongst processes?
- Which of a particular process's pages to keep in memory?
- A poor choice can lead to horrible performance
  - cost of paging: disk much, much slower than memory

Page Replacement

When a page fault occurs, the OS loads the faulted page from disk into a page frame of physical memory

At some point, the process used all of the page frames it is allowed to use

- This is likely (much) less than all of available memory

When this happens, the OS must replace a page for each page faulted in

- It must evict a page to free up a page frame

The page replacement algorithm determines how this is done

- Greatly affect performance of paging (virtual memory)
- Also called page eviction policies

Locality

All paging schemes depend on locality

- Processes reference pages in localized patterns

Temporal locality

- Locations referenced recently likely to be referenced again

Spatial locality

- Locations near recently referenced locations are likely to be referenced soon

While the cost of paging is high, if it is infrequent enough it is acceptable

- Processes usually exhibit both kinds of locality during a run, making paging practical

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First-In First-Out (FIFO)

Evict oldest fetched page in system

Example: suppose we have 3 page frames, 5 virtual pages and the following reference string: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

FIFO Page replacement: 9 page faults

Reference	1	2	3	4	1	2	5	1	2	3	4	5
Frame 0	1			4			5					
Frame 1		2			1					3		
Frame 2			3			2					4	

First-In First-Out (FIFO)

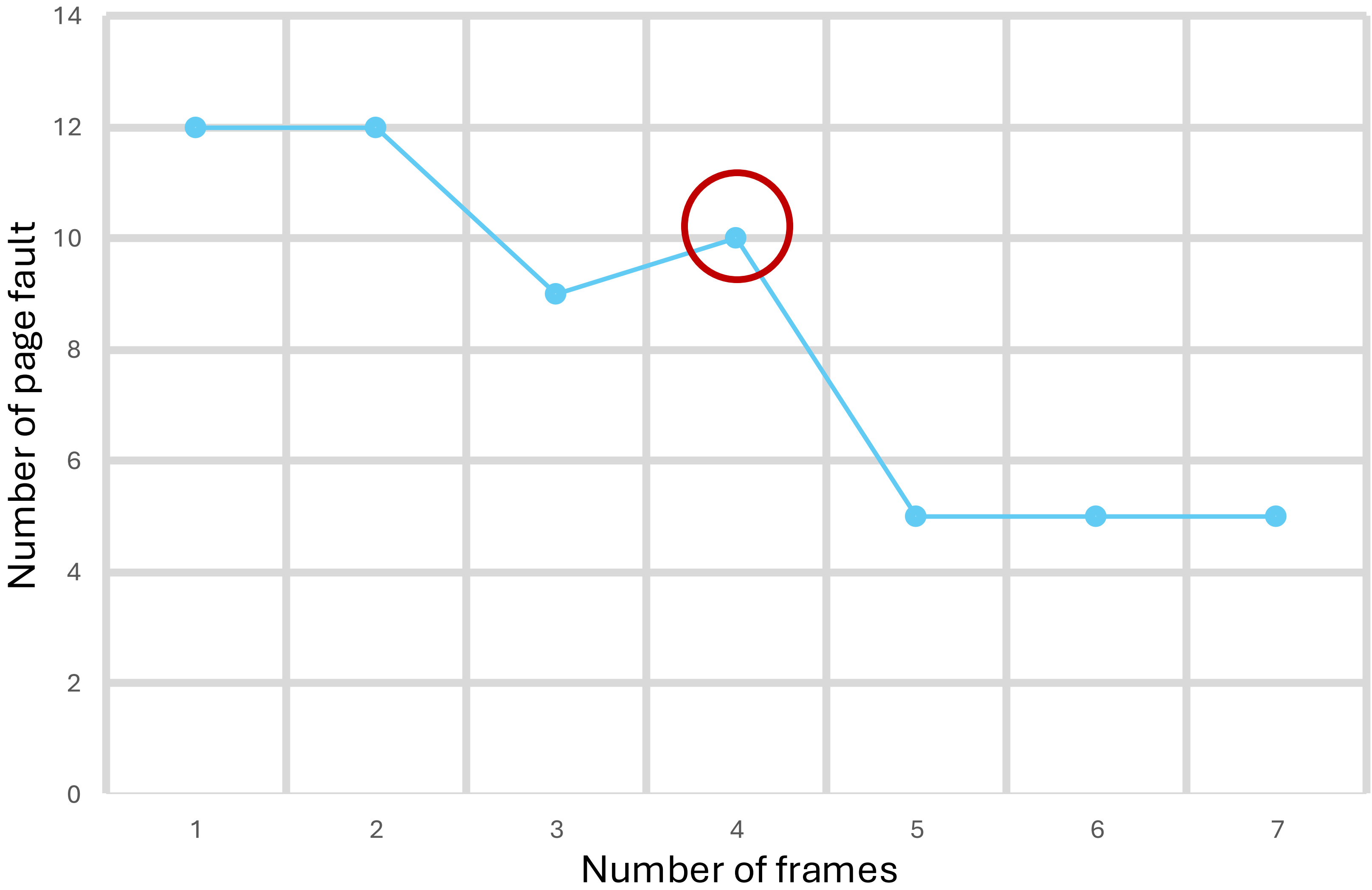
Evict oldest fetched page in system

Example: suppose we have 4 page frames, 5 virtual pages and the following reference string: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

FIFO Page replacement: 10 page faults

Reference	1	2	3	4	1	2	5	1	2	3	4	5
Frame 0	1						5				4	
Frame 1		2						1				5
Frame 2			3						2			
Frame 4				4						3		

Belady’s Anomaly



More physical memory doesn’t always mean fewer faults

Optimal Page Replacement

What is optimal (if you knew the future)?

- Replace page that will not be used for longest period of time

Example: suppose we have 4 page frames, 5 virtual pages and the same reference string: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

Optimal Page replacement: 6 page faults

Reference	1	2	3	4	1	2	5	1	2	3	4	5
Frame 0	1										4	
Frame 1		2										
Frame 2			3									
Frame 4				4			5					

Belady’s Algorithm

Known as the optimal page replacement algorithm

- Rationale: the best page to evict is the one never touched again
- Never is a long time, so picking the page closest to “never” is the next best thing
- Proved by Belady

Problem: Have to predict the future

Why is Belady’s useful then? Use it as a yardstick

- Compare page replacement algorithms with the optimal to gauge room for improvement
- If optimal is not much better, then algorithm is pretty good
- If optimal is much better, then algorithm could use some work
  - Random replacement is often the lower bound

Least Recently Used (LRU)

Approximate optimal with least recently used

- Because past often predicts the future
- On replacement, evict the page that has not been used for the longest time in the past (Belady's: future)

LRU Page replacement: 8 page faults

Reference	1	2	3	4	1	2	5	1	2	3	4	5
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LRU Page replacement: 8 page faults

Problem 1: Can be pessimal – example?

- Looping over memory (then want MRU eviction)

Problem 2: How to implement?

Strawman LRP Implementation

Stamp PTEs with timer value

- E.g., CPU has cycle counter
- Automatically writes value to PTE on each page access
- Scan page table to find oldest counter value = LRU page
- Problem: Would double memory traffic!

Keep doubly-linked list of pages

- On access remove page, place at tail of list
- Problem: again, very expensive

What to do?

- Just approximate LRU, don't try to do it exactly

Clock Algorithm

Use accessed bit supported by most hardware

- E.g., Pentium will write 1 to A bit in PTE on first access
- Software managed TLBs like MIPS can do the same

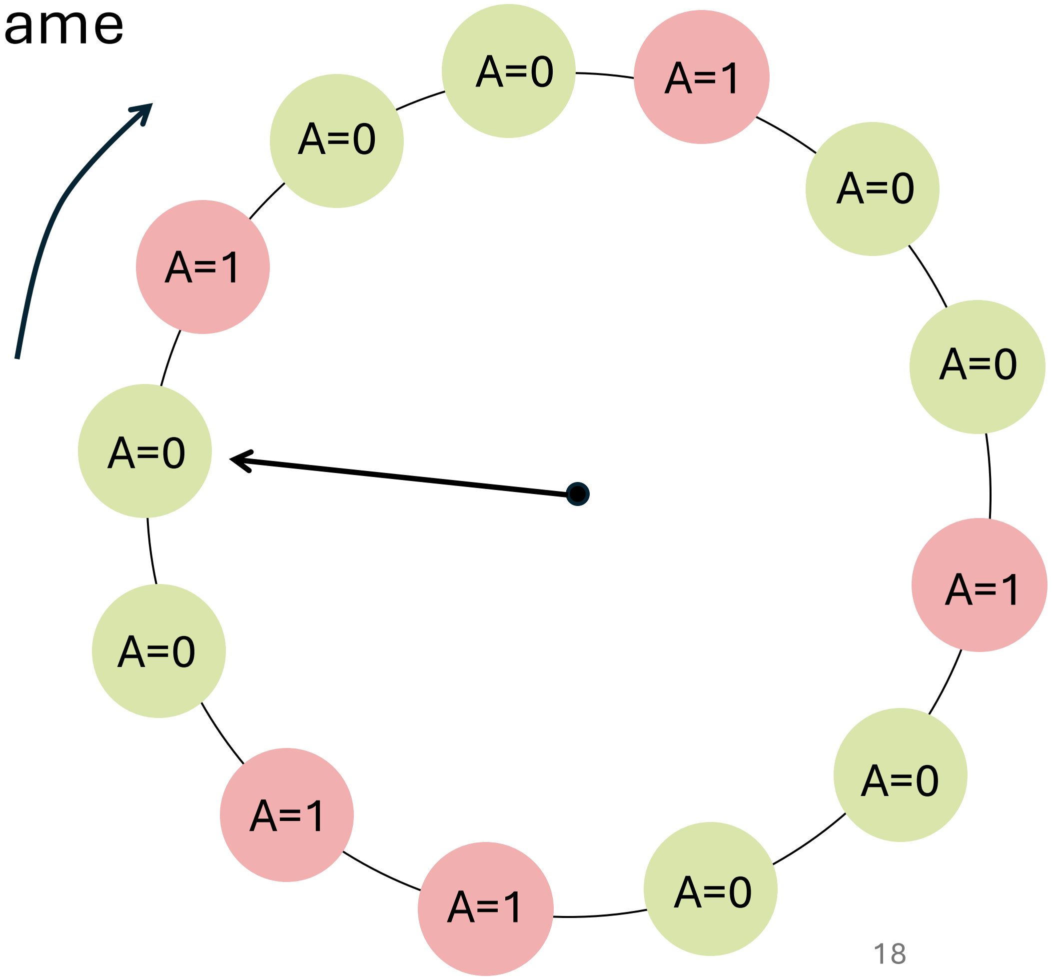
Do FIFO but skip accessed pages

Keep pages in circular FIFO list

Scan:

- page's A bit = 1, set to 0 & skip
- else if A = 0, evict

A.k.a. second-chance replacement



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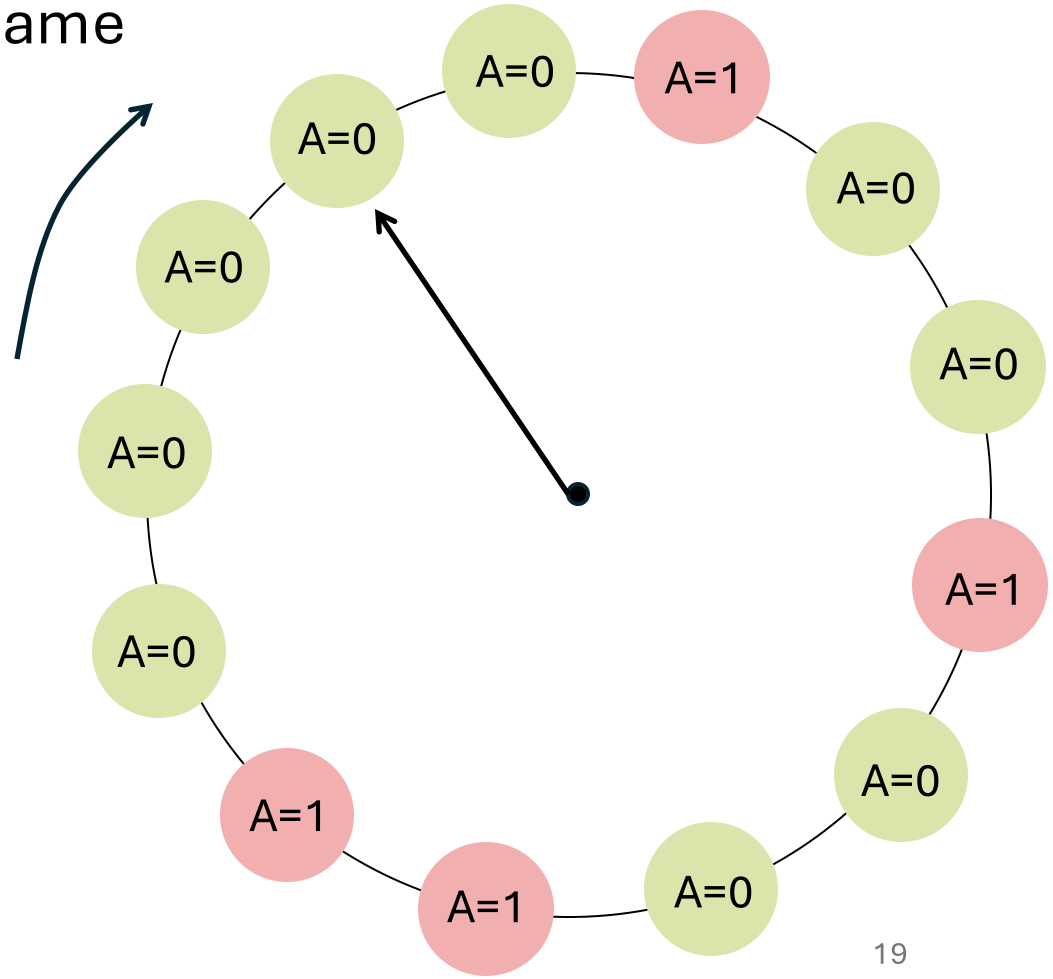

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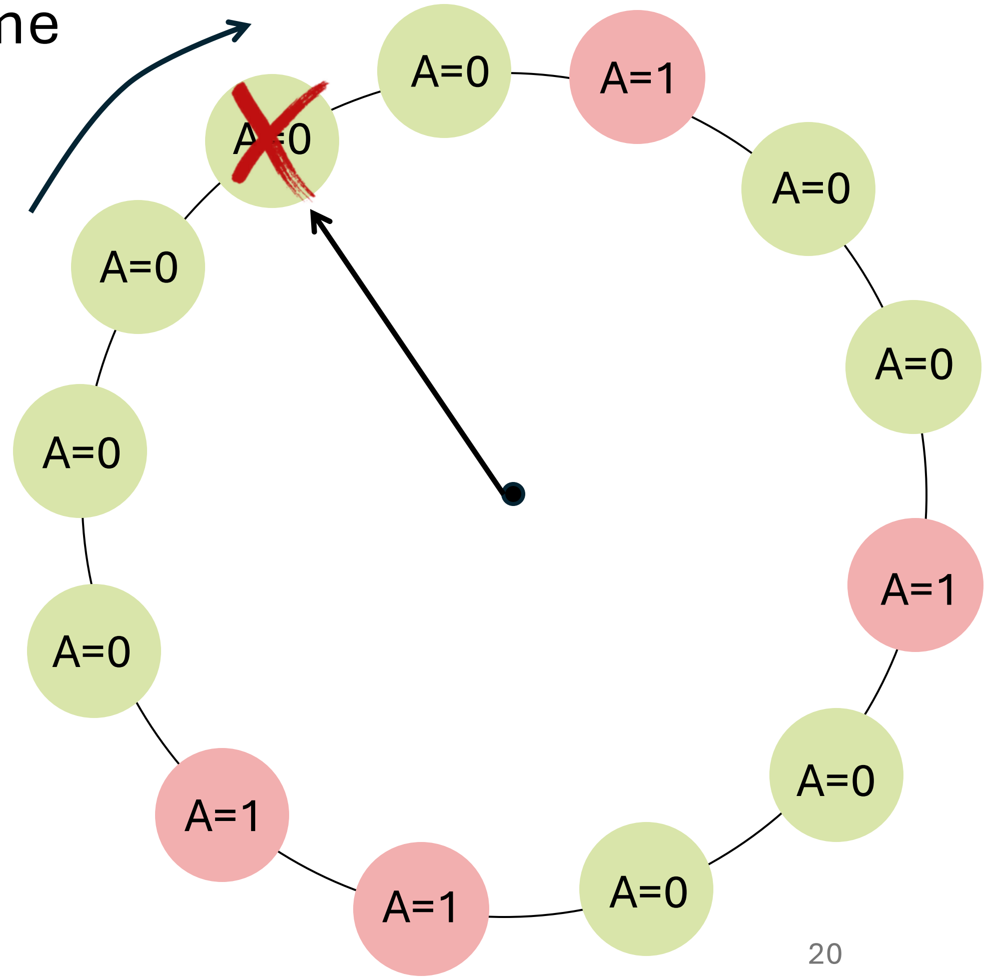


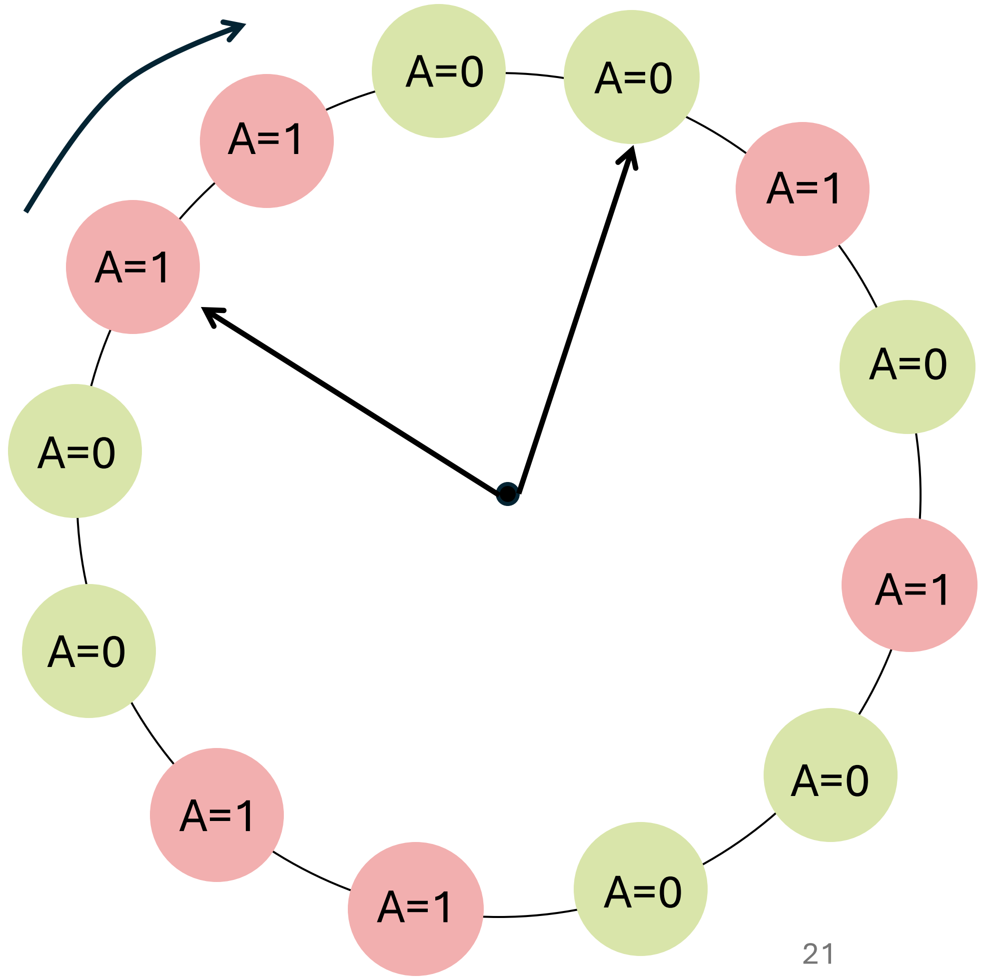
Clock Algorithm (2)

Large memory may be a problem

- Most pages referenced in long interval

Add a second clock hand

- Two hands move in lockstep
- Leading hand clears A bits
- Trailing hand evicts pages with A=0



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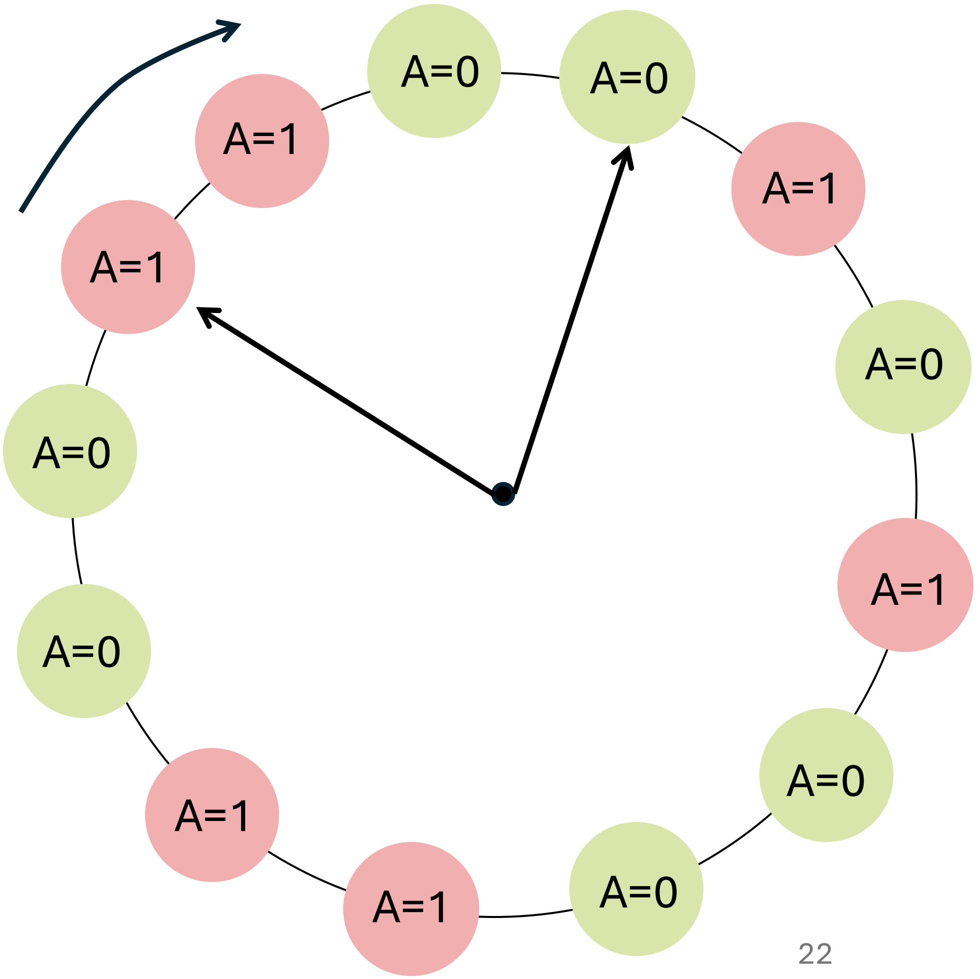

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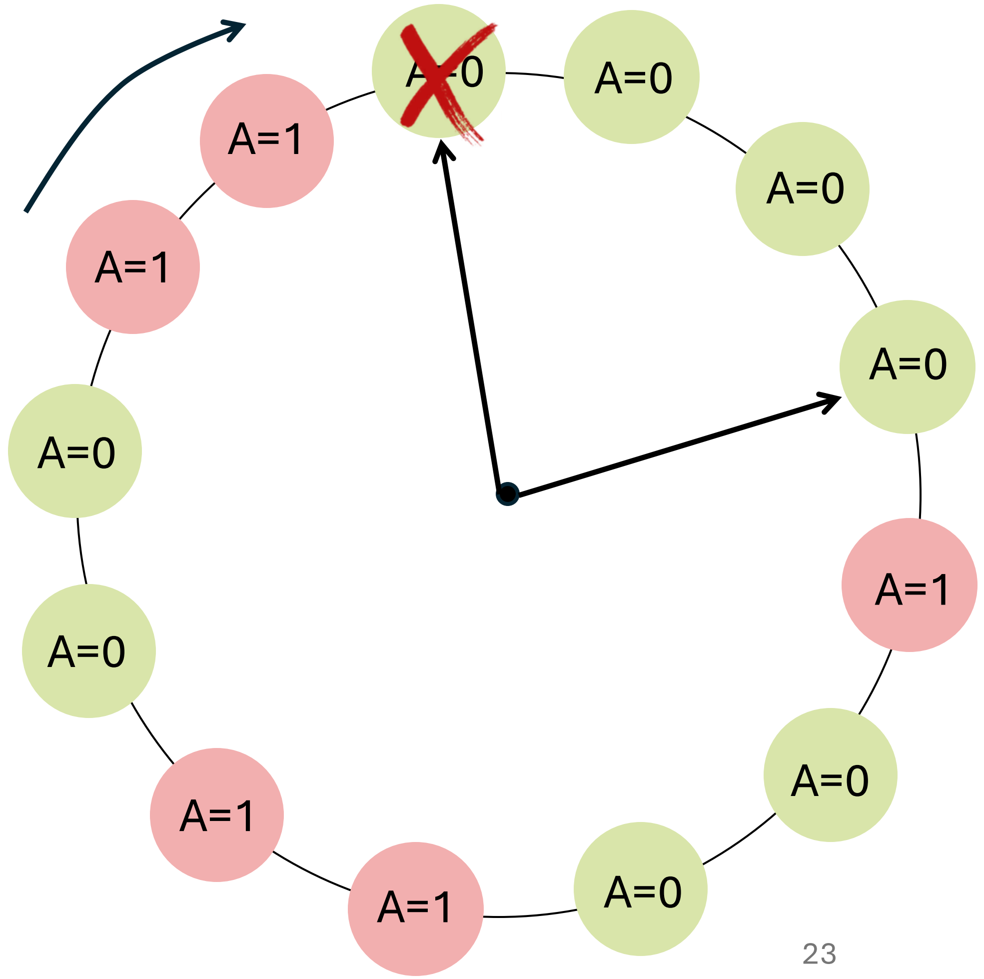

Other Replacement Algorithms

Random eviction

- Dirt simple to implement
- Not overly horrible (avoids Belady & pathological cases)

LFU (least frequently used) eviction

- Instead of just A bit, count # times each page accessed
- Least frequently accessed must not be very useful (or maybe was just brought in and is about to be used)
- Decay usage counts over time (for pages that fall out of usage)

MFU (most frequently used) algorithm

- Because page with the smallest count was probably just brought in and has yet to be used

Neither LFU nor MFU used very commonly

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Fixed vs. Variable Space

How to determine how much memory to give to each process?

Fixed space algorithms

- Each process is given a limit of pages it can use
- When it reaches the limit, it replaces from its own pages
  - Local replacement
- Some processes may do well while others suffer

Variable space algorithms

- Process' set of pages grows and shrinks dynamically
- Global replacement
  - One process can ruin it for the rest

Working Set Model

A working set of a process is used to model the dynamic locality of its memory usage

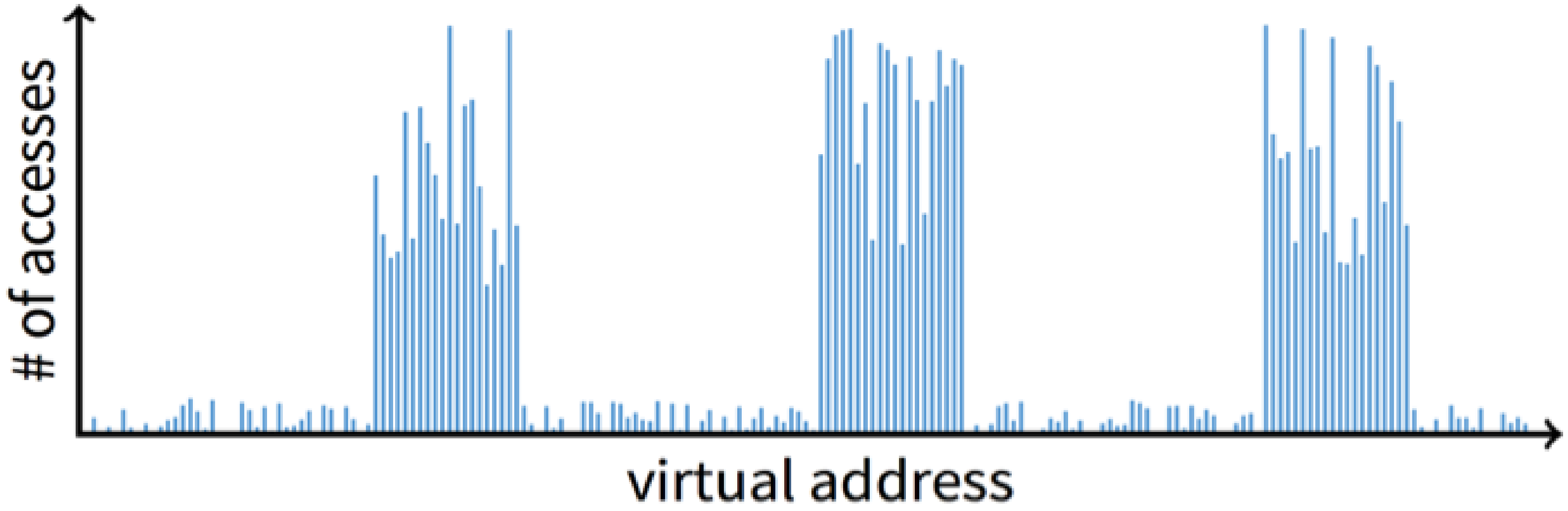
- Defined by Peter Denning in 60s, published at the first SOSP conference

Definition

- $WS(t, w)$ = {pages P such that P was referenced in the time interval (t, t-w)}
- t – time, w – working set window (measured in page refs)

A page is in the working set (WS) only if it was referenced in the last w references

Working Set Model



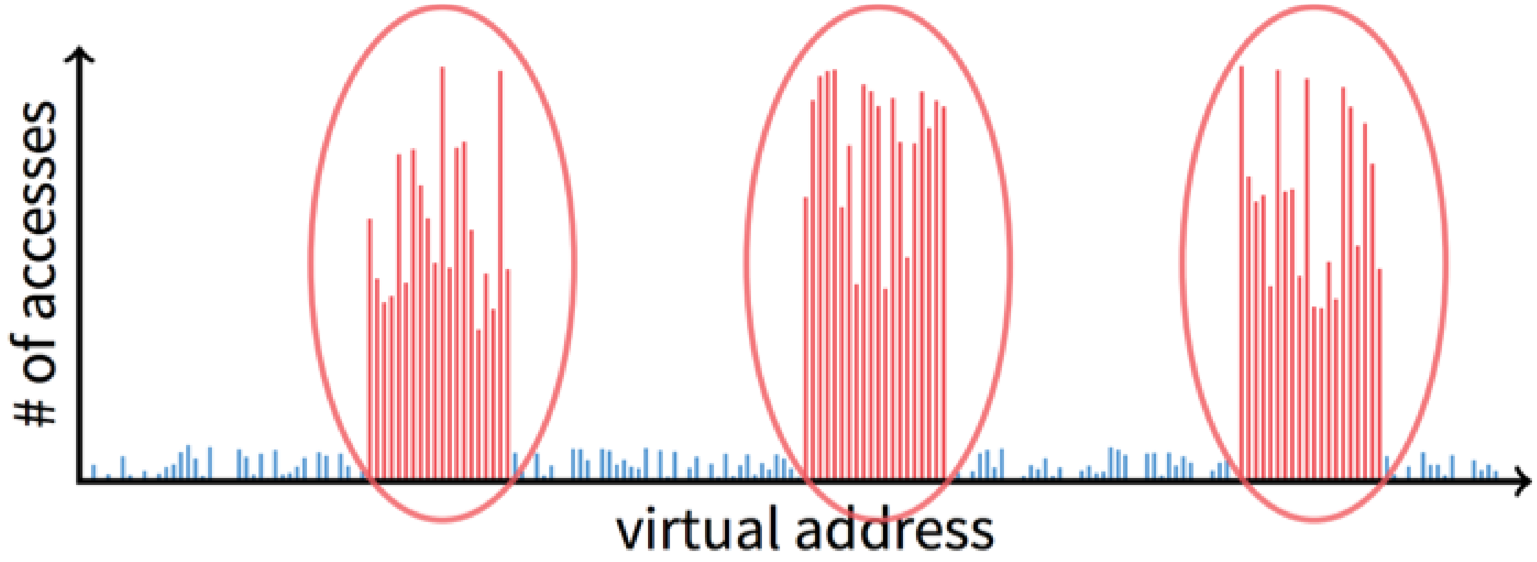
Disk much, much slower than memory

- Goal: run at memory speed, not disk speed

80/20 rule: 20% of memory gets 80% of memory accesses

- Keep the hot 20% in memory
- Keep the cold 80% on disk

Working Set Model



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Working Set Size

The working set size is the # of unique pages in the working set

- The number of pages referenced in the interval $(t, t - w)$

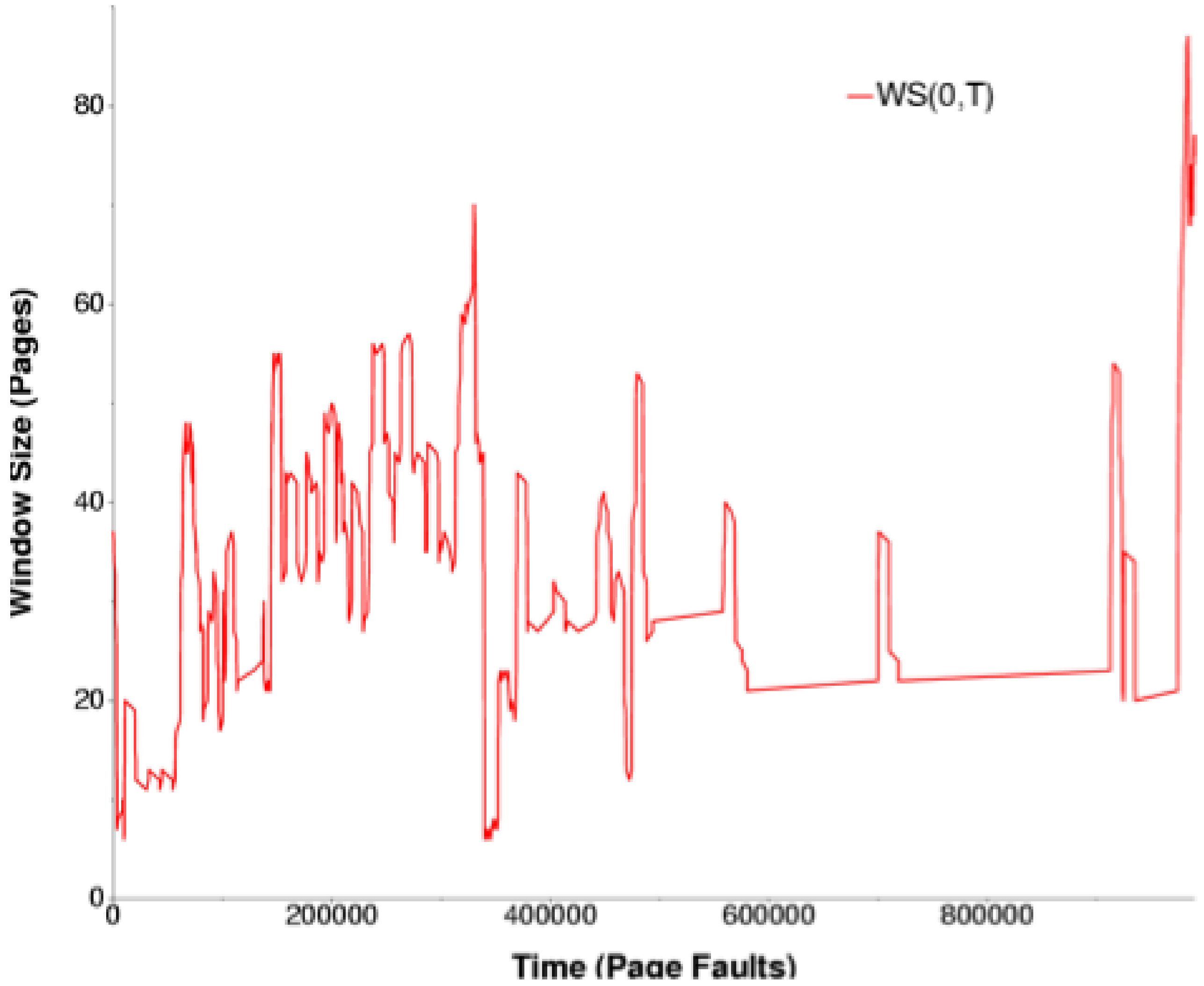
The working set size changes with program locality

- During periods of poor locality, you reference more pages
- Within that period of time, the working set size is larger

Intuitively, want the working set to be the set of pages a process needs in memory to prevent heavy faulting

- Each process has a param w that determines a working set with few faults
- Denning: Don’t run a process unless working set is in memory

Example: gcc Working Set



Working Set Problems

Problems

- How do we determine w?
- How do we know when the working set changes?

Too hard to answer

- So, working set is not used in practice as a page replacement algorithm

However, it is still used as an abstraction

- The intuition is still valid
- When people ask, “How much memory does Firefox need?”, they are in effect asking for the size of Firefox’s working set

Page Fault Frequency (PFF)

Page Fault Frequency (PFF) is a variable space algorithm that uses a more ad-hoc approach

- Monitor the fault rate for each process
- If the fault rate is above a high threshold, give it more memory
  - So that it faults less
  - But not always (FIFO, Belady's Anomaly)
- If the fault rate is below a low threshold, take away memory
  - Should fault more
  - But not always

Hard to use PFF to distinguish between changes in locality and changes in size of working set

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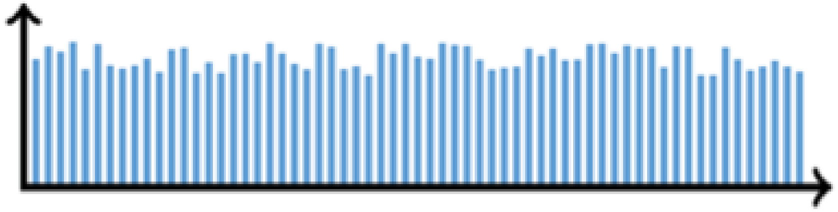
Thrashing

Page replacement algorithms avoid thrashing

- When OS spent most of the time in paging data back and forth from disk
- Little time spent doing useful work (making progress)
- In this situation, the system is overcommitted
    - No idea which pages should be in memory to reduce faults
    - Ex: Running Windows95 with 4 MB of memory...

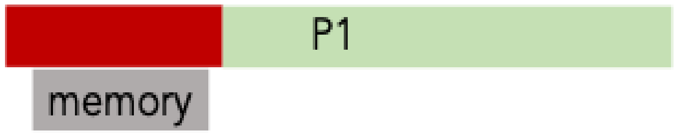
Reason for Thrashing

Access pattern has no temporal locality (past $\not\approx$ future)

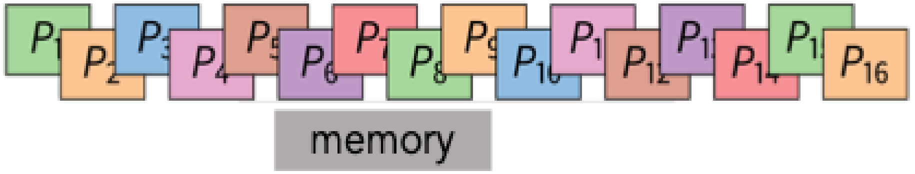


80/20 rule has broken

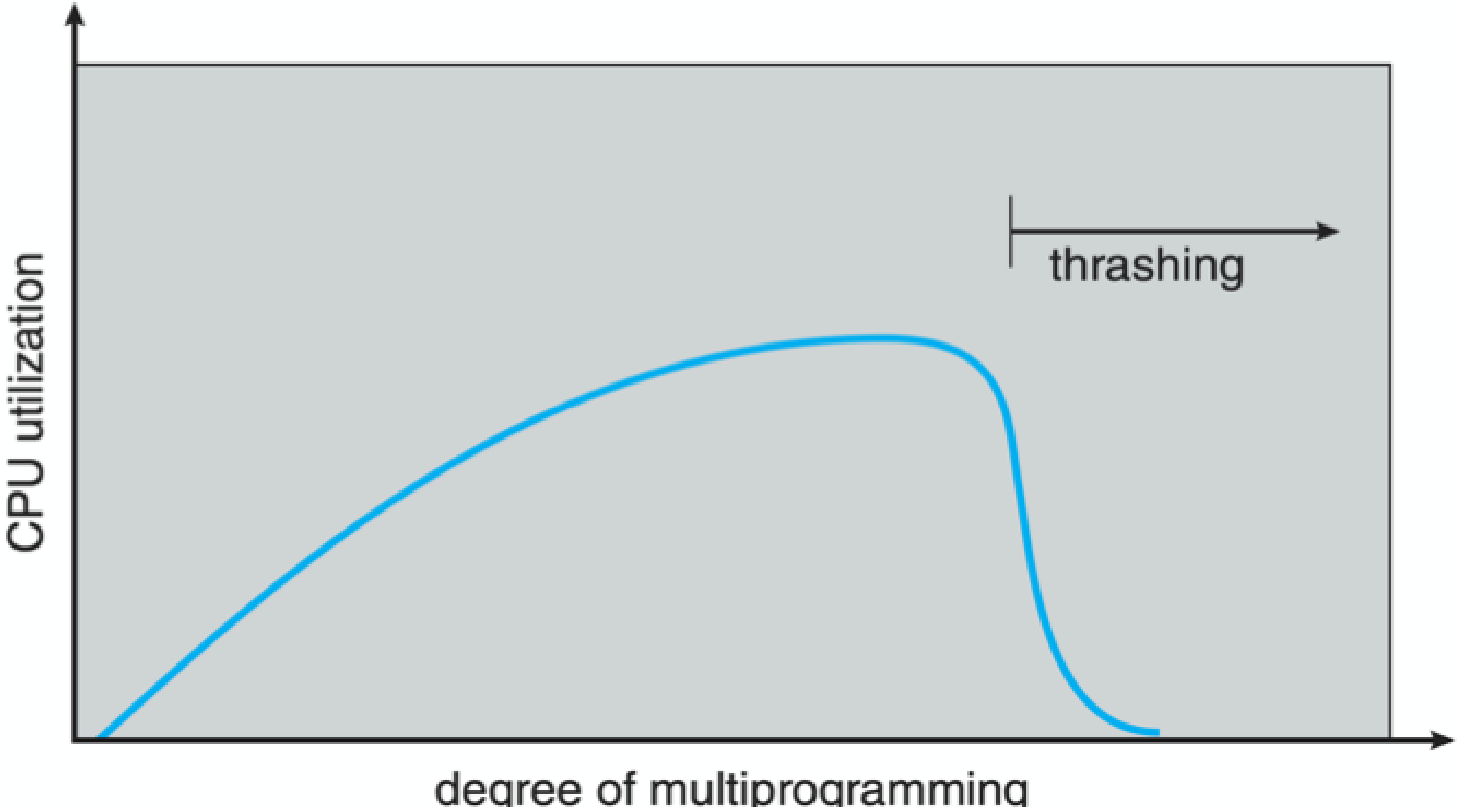
Hot memory does not fit in physical memory



Each process fits individually, but too many for system



Thrashing & Multiprogramming



Dealing with Thrashing

Only run processes if memory requirements can be satisfied

- Thrashing viewed from a caching perspective: given locality of reference, how big a cache does the process need?
- Or: how much memory does the process need in order to make reasonable progress (its working set)

Swapping – write out all pages of a process

Buy more memory…

Summary

Page replacement algorithms

- Belady's – optimal replacement (minimum # of faults)
- FIFO – replace page loaded furthest in past
- LRU – replace page referenced furthest in past
  - Approximate using PTE reference bit
- LRU Clock – replace page that is "old enough"
- Working Set – keep the set of pages in memory that has minimal fault rate
- Page Fault Frequency – grow/shrink page set as a function of fault rate

Multiprogramming

- Should a process replace its own page, or that of another?

Next Time…

Read Chapter 14,17